A COMPARISON OF ANALYTICALLY AND EXPERIMENTALLY DETERMINED WAVE PARAMETERS ON THE LIQUID INTERFACE OF THE AIRCORE OF A SWIRL ATOMISER

J. J. Chinn*, D. Cooper* and A. J. Yule[o]

*Department of Mechanical, Aerospace and Civil Engineering. UMIST. Sackville Street. Manchester. M60 1QD. England. Contact: j.chinn@umist.ac.uk, 0044 (0)161 306 3733.
[o]Department of Mechanical Engineering. Newton Building. University of Salford. Manchester. England. M5 4WT.

ABSTRACT

In 2003 Chinn [1] presented an analytical derivation of a one-dimensional wave equation applicable to the waves on the surface of the aircore of a swirl atomiser, by analogy to the similar, text-book, one-dimensional wave equation derivation for sea-going gravity waves. In addition Chinn [2] presented an analytical derivation of the weir flow analogy to the outlet flow in a swirl atomiser. It was indicated that the expression for the critical velocity at the outlet, from the weir flow treatment, was equal to the wave phase velocity derived in Chinn [1].

In 2001 Cooper and Yule [3] described their experimental discovery of different wave phenomena occurring on the surface of the aircore of a swirl atomiser. In particular Cooper and Yule describe both a standing wave within the body of the swirl atomizer and corresponding waves occurring on the issuing conical liquid sheet at the exit. The present work makes a quantitative comparison between these analytical and experimental findings with remarkably accurate results.

INTRODUCTION

The motivation for the present work came about through a desire to better understand the seminal analytical and experimental work of Binnie et. al. [5, 6] who performed similar types of analytical work on swirling flows through vortex tubes and experimental work on whirlpools. However Binnie et. al. do not mention any actual discovery of wave phenomenon. Crapper et. al. [4] performed an important contribution to analytical wave treatments on rotating sheets. In all of these works there has been an endeavour to better understand the complexity of the flow physics of swirling flows with wave motion using simple analytical techniques. Rayleigh [7], writing in 1916 states "So much of meteorology depends ultimately upon the dynamics of revolving fluid that it is desirable to formulate as clearly as possible such simple conclusions as are within our reach, in the hope that they may assist our judgement when an exact analysis seems intractable." Nowadays, of course, such 'exact' analysis are increasingly more readily available in the form of computational fluid dynamics techniques. There is however, still room for these analytical techniques for, in their derivation, one does gain a better insight into the forces and flow physics of swirling flow with waves.

VISUALISATIONS

Cooper et. al. [8] postulated that the waves occurring on the conical liquid sheet, issuing from the outlet of a swirl atomiser, originate from further upstream, on the surface of the aircore within the body of the atomiser. Later work of Cooper and Yule [3] appears to prove this.

As part of their work, Cooper and Yule [3] used an oversize, two-inlet, Perspex model swirl atomiser, water as the operating fluid and a high-speed Kodak Ecstapo Motion Analyser and data capture card for a personal computer. Amongst their finding were standing waves within the swirl chamber of the atomiser. In particular, for a flow rate of 0.428 lt/s observations were made of a phenomenon whereby localised regions of the aircore were seen to expand and contract with a regular period in a form of pulsating manner. The expanding-contracting regions appeared in two locations on the aircore-liquid interface, spaced approximately 48 mm apart. Examination of a number of the video frames indicated that the period between expansion and contraction to be approximately 0.038 seconds.

Figure 1 shows two still images taken 0.038 seconds apart and indicates this standing wave phenomenon.

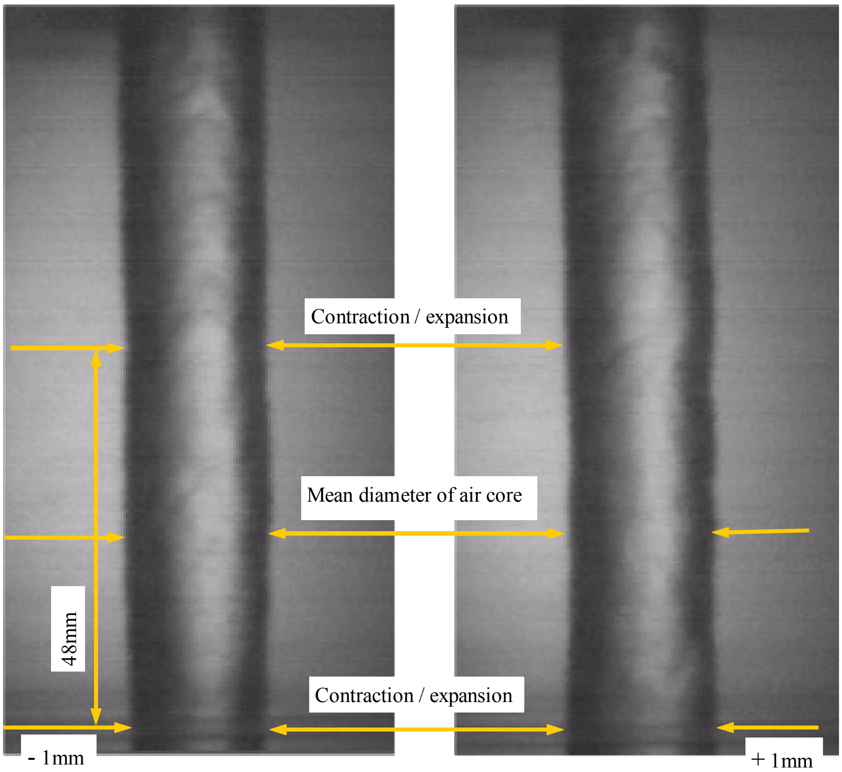

Figure 1. Two video frames of air core 0.038 sec. apart, for a flow rate of 0.468 lt/s

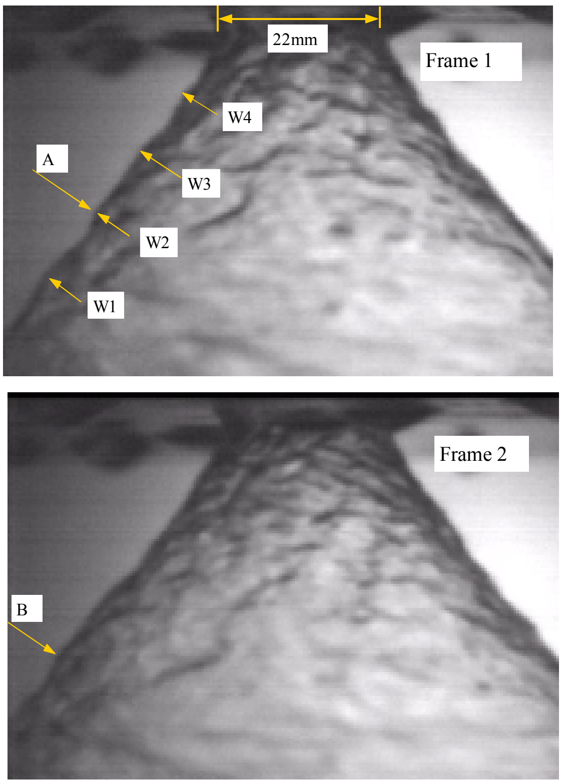

Figure 2. Waves on the spray cone for a flow rate of 0.428 lt/s

Figure 2 is a view of the spray cone at 0.428 lt/s, on the left hand edge can be seen a number of waves: W1 to W4. The distance (λ) between W1 and W2 is 10.5mm, between W2 and W3 is 9.4mm and between W3 and W4 is 8.7mm which indicates that the spray cone is expanding. The time interval between frames is 2.6ms and the distance travelled by W2 in this time (from A to B) is 9.5mm giving a linear velocity of 3.7m/s. Applying the same measurements to the other three waves gave a phase velocities of 2.4m/s for W1, 3.9m/s for W3 and 3.6m/s for W4. This gives an average wave phase velocity (χ) of 3.3±0.8m/s.

If the average distance between waves is taken as 9.5mm at an average velocity of 3.3m/s then the time interval (τ) between consecutive waves is 0.029 seconds which is of the same order as the 0.038 seconds time interval recorded between the localised expansion-contraction of the aircore within the swirl chamber. It is believed that these waves are a result of the varying liquid film thickness in the exit orifice which itself is a result of the local expansion-contraction of regions on the aircore.

The aircore diameter within the outlet was measured from a number of still images with account taken of the refraction of the water. The flow within the outlet had become quite chaotic and turbulent at this juncture. However it can be categorically stated that the radius falls between the limits of 8.5 mm and 9.3 mm: r_{ac} = 8.9±0.4mm.

ANALYTICAL DERIVATIONS

Low Amplitude Rotary Force Waves

Several advance text books on water waves (e.g. Crapper [9]) provide a mathematical derivation of a wave equation for long, shallow water gravity waves. This derivation uses the catesian coordinate system and is based on the Bernoulli equation for inviscid flows. The theory assumes an asymptotic power series for the wave form. Chinn [1] used this theory and by substituting a rotary force for that of gravity, created a similar wave equation for thin-walled rotating sheets. This wave equation, together with the expression derived for the wave phase velocity as given as

$$\frac{\partial^2 \eta}{\partial t^2} = \chi^2 \frac{\partial^2 \eta}{\partial x^2} \qquad \text{where} \quad \chi = \sqrt{\frac{c^2\left(r_w^2 - r_{ac}^2\right)}{2r_{ac}^4}}.$$

Here c is the vorticity constant: wr = c = constant, for an inviscid, axisymmetric treatment.

The Principle of Maximum Flow

The principle of maximum flow was used by a number of early workers attempting to discern the aircore diameter, discharge coefficient and spray cone angle for prototype, drawing board, atomisers. Principle among these was the much quoted work of Giffen and Muraszew [10]. Essentially the principle of maximum flow, in cylindrical coordinates with a rotary force is similar to the flow occurring over the crest of a weir, in cartesian coordinates under gravity. The principle basically states that there is an equilibrium liquid sheet thickness and exit velocity at the outlet commensurate with the rotary force of the swirling liquid. Chinn [2], in developing the principle of maximum flow for a swirl atomiser, has shown that what is remarkable is that the critical, or optimum velocity at the outlet transpires to be equal to the wave phase velocity derived for the low amplitude rotary force waves and is given by

$$u_c = \sqrt{\frac{c^2\left(r_w^2 - r_{ac}^2\right)}{2r_{ac}^4}}.$$

COMPARISON

Earlier it was indicated that the wave phase velocity calculated from measurements taken from the visualisations of the actual spray was

χ = 3.3±0.8m/s.

In order to compare the theoretical wave phase velocity a value must first be determine for the vorticity constant c. The vorticity constant is given as c = wr. It is easy to establish values for both w and r at the inlets of the atomizer, as advocated by Bayvel and Orzechowski [11]: $c = wr = w_i R$. The mean inlet tangential velocity will be given simply by dividing the supply volumetric flow rate by the cross-sectional area of the inlets:

$$w_i = \frac{Q}{d_p} = \frac{4.28\times10^{-4}}{2\times11\times10^{-3}\times12\times10^{-3}} = 1.62 \text{ m/s}$$

The radius at which the inlet tangential velocity, w_i , acts is

$$R = r_w - w_p/2 = 36.36 \text{ x } 10^{-3} - 6.0 \text{ x}10^{-3} = 30.36 \text{ x}10^{-3} \text{ m.}$$

The vorticity constant is therefore

$$c = wr = w_i R = 1.62 \text{ x } 30.36 \text{ x}10^{-3} = 4.92 \text{ x}10^{-2} \text{ m}^2\text{/s.}$$

Using the measured aircore diameter of $r_{ac} = 8.9\pm0.4$mm in the expression indicated earlier for the wave phase velocity thus gives

$$\chi = \sqrt{\frac{c^2\left(r_w^2 - r_{ac}^2\right)}{2r_{ac}^4}} = 2.8 \pm 0.5 \text{ m/s}$$

Within the limits of experimental error, for both the measured phase velocity and the measured aircore radius, there is a definite overlap between the two values.

CONCLUSION

The visualisations indicate that the pulsating, standing waves on the air core within the swirl atomiser give rise to the waves on the conical liquid sheet issuing from the exit, they are of a similar periodicity. The thin film rotary force wave theory predicts the phase velocity of the waves at the outlet. The weir flow analogy concurs with this result. In other words there appears to be a link between these theories in the same way as there is a link between the swirl chamber standing waves and the outlet progressive waves. The present work makes a quantitative comparison between these analytical and experimental results mainly for the purpose of adding weight to the validity of the mathematical analytical work. The limitation is that there is no mathematical link between the two theories. The two theories are currently disparate and some deep thought, which would give great physical insight, would be necessary to unite them.

No prediction can be made of the wave length, period or amplitude with the current two dimensional wave theory. More sophisticated three-dimensional analytical techniques may well suggest ways in which these parameters can be predicted.

NOMENCLATURE

Outlet wall radius	$r_w = 11$mm	($11.0 \text{ x}10^{-3}$ m)
Measure outlet aircore radius	$r_{ac} = 9$ mm	($9.0 \text{ x}10^{-3}$ m)
Volumetric Flow Rate	$Q = 0.428$ lt/s	($4.28 \text{ x}10^{-4}$ m^3/s)
Swirl Chamber Radius	$r_s = 36.36$ mm	($36.36 \text{ x } 10^{-3}$ m)
Inlet Height	$h_p = 11$ mm	($11.0 \text{ x}10^{-3}$ m)
Inlet Width	$w_p = 12$ mm	($12.0 \text{ x}10^{-3}$ m)
Inlet cross-sectional area	$d_p = 2.64\text{x}10^{-4}$ m^2	
Vorticity constant	$c = 4.92 \text{ x}10^{-2}$ m^2/s	
Axial coordinate	x	(m)
Radial coordinate	r	(m)
Tangential coordinate	θ	(radians)
Axial Velocity	u	(m/s)
Radial Velocity	v	(m/s)
Tangential Velocity	w	(m/s)

Mean Inlet Radius	R $(= r_w - w_p/2)$	(m)
Time	t	(s)
Wave height	η	(m)
Wave Phase Velocity	χ	(m/s)
Wave Period	τ	(s)
Wave Length	λ	(m)

REFERENCES

(1) J. J. Chinn, The Analogy between Waves on the Surface of the Aircore of a swirl Atomiser and Long, Shallow Water Gravity Waves. *Proc. ICLASS-Europe. Sorrento, Italy*. 2003.

(2) J. J. Chinn , The Analogy between Swirl Atomizer and Weir Flow: The Principle of Maximum Flow. *Proc. ICLASS-Europe. Sorrento, Italy*. 2003.

(3) D. Cooper and A. J. Yule, Waves on the Air Core/Liquid Interface of a Pressure Swirl Atomiser. *Proc. ILASS-Europe. Zurich, Switzerland.* 2001.

(4) G. D. Crapper, N. Dombrowski and G. A. D. Pyott, Kelvin-Helmholtz Wave Growth on Cylindrical Liquid Sheets. *Journal of Fluid Mechanics*. Vol. 68. Part 3. pp 497- 502.

(5) A. M. Binnie and G. A. Hooking, Laboratory Experiments on Whirlpools. *Proceedings of the Royal Society of London*. Series A. Vol. 194A. pp 398-415. 1948.

(6) A. M. Binnie, The Passage of a Perfect Fluid Through a Critical Cross-Section or 'Throat'. *Proceedings of the Royal Society of London*. Series A. Vol. 197A. pp 545-555. 1949.

(7) J. W. S. Rayleigh, On the Dynamics of Revolving Fluids. *Proceedings of the Royal Society of London.* Series A. Vol. 93. pp 148-154.

(8) D. Cooper, J. J. Chinn and A. J. Yule, Experimental Measurements and Computational Predictions of the Internal Flow Field in a Pressure Swirl Atomiser. *Proc. ICLASS-Europe. Toulouse.* 1999.

(9) G. D. Crapper, *Introduction to Water Waves*, Ellis Horwood. ISBN 0-85312-487-6. 1984.

(10) E. Giffen and A. Muraszew, *Atomization of Liquid Fuels*. Chapman and Hall. 1953.

(11) L. P. Bayvel and Z. Orzechowski, *Liquid Atomization*. Taylor and Francis. ISBN 0-89116-959-8. 1993.

ACKNOWLEDGEMENTS

The authors are grateful for support from the following organisations:

- United Kingdom Engineering and Physical Science Research Council
- The Royal Society of Engineering
- Glaxo-Welcome, Smith-Kline Beecham Pharmaceuticals
- Ansul-Wormald Sprinkler Systems