THE NUMERICS OF THE SWIRL ATOMIZER

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ABSTRACT

Pressure swirl atomizer flow is inherently complicated. The primary internal flow regime incorporates two-phases with sharp velocity and pressure gradients. The flow is turbulent and takes place on a scale where density and viscosity effects are important considerations. Secondary flow effects include annular vortex formation with corresponding surface waves on the air core. Velocity components at the outlet may be supersonic. Surface tension effects, both on the air core and on the resultant ligaments and droplets, are important considerations for predicting spray formation. This level of flow complexity is difficult to comprehend and to predict, and to quantify, for any given atomizer design, working fluid and working environment. Dimensionless numerical quantities, or "Numerics", can be helpful aids to comprehension and quantification. The numerics include: Reynolds number, the ratio of inertia to viscous forces; Weber number, the ratio of inertial to surface tension forces (both on the air core and on the droplets); Froude number, a ratio of gravity to bulk flow effects and many others. This article covers the dimensionless quantities of Froude Number, Mach Number, Swirl Number, Reynold number and Weber number, both on the air core and on a droplet.

Froude Number, Fr

The Froude number shows the effects of gravity in comparison to the energy of the bulk flow; and is given as the square root of the ratio of kinetic energy to a gravitational potential energy. If one considers a swirl atomizer used with its axis vertically; then Fr may be defined using the ratio of the axial velocity head term to the gravity head term, from the Bernoulli Equation, per unit mass of liquid. The axial velocity within the outlet of a swirl atomizer is approximated, by dividing the volume flow, Q, by the annulus of fluid between the air core r_{oac} and the outlet wall, r_o. This is given by [1], as

$$u = \frac{Q}{\pi \left(r_o^2 - r_{oac}^2 \right)}. \tag{1}$$

The length scale used in the gravity head term may be taken to be the length of the outlet. This is typically of the order of the diameter of the outlet, [2]:

$$h \sim 2r_o \tag{2}$$

thus

$$Fr = \sqrt{\frac{\text{Kinetic Energy}}{\text{Potential Energy}}} = \sqrt{\frac{u^2}{2hg}} = \tag{3}$$

$$\sqrt{\frac{Q^2}{2\pi^2 \left(r_o^2 - r_{oac}^2 \right)^2 \times 2r_o g}} = \frac{Q}{2\pi \left(r_o^2 - r_{oac}^2 \right)\sqrt{r_o g}}.$$

A Froude number of order unity (Fr ~ 1) indicates that neither gravitational or kinetic forces dominate. As an example of Fr, consider the large atomizer used by DeKeukalaere [3], which was run at a low operating pressure ($\Delta p = 15$kPa). This gave $Q = 2 \times 10^{-4}$ m³/s and $r_{oac} = 3.4$mm. With $r_o = 5.5$mm this gives Fr = 7.5. Which indicates that kinetic energy dominates over gravitational potential energy. DeKeukelaere [3] noticed a slight increase in wall pressure in the swirl chamber, near to the convergence, when the atomizer was run vertically, at this low velocity, and attributed the rise to the effect of gravity. As a converse example, consider one of the small, high speed, atomizers of [2]; $r_o = 1.0$mm, $r_{oac} = 0.79$mm, $Q = 4.1 \times 10^{-5}$m³/s. Gives Fr = 175, which clearly indicates the dominance of kinetic energy over gravitational potential energy, for the majority of practical applications.

Weber Number, We, On the Air Core

The Weber number is defined as a ratio of inertial force over surface tension force

$$We = \frac{\text{Inertial Force}}{\text{Surface Tension Force}} = \frac{\rho V^2 L}{\gamma} \tag{4}$$

where ρ is the liquid density, V is some characteristic velocity, L is some characteristic length and γ is the surface tension (force per unit length). Consider the situation on the air core of a swirl atomizer. The inertial force, acting in an

outward radial direction, may be given by the angular acceleration, w_{oac}^2/r_{oac}, multiplied by the mass of a fluid element, $dm = \rho\, r_{oac}\, \delta\theta\, \delta r\, dx$, where, for We = 1, δr is the difference that surface tension may make to the air core size. The opposing surface tension force, acting in an inward radial direction, can be discerned with reference to Fig.(1). This is $2\gamma \sin(\delta\theta) \sim 2\gamma\, \delta\theta$, for small angle in radian measure. Thus

$$\mathrm{We} = \frac{\dfrac{w_{oac}^2}{r_{oac}}\rho r_{oac}\delta\theta\,\delta r\,dx}{2\gamma\,\delta\theta\,dx} = \frac{w_{oac}^2\rho\delta r}{2\gamma} = \left(\frac{Qr_s}{A_i r_{oac}}\right)^2 \frac{\rho\delta r}{2\gamma} \qquad (5)$$

where the swirl velocity on the air core, w_{oac}, is taken as $Qr_s/A_i r_{oac}$, [1], where r_s is the radius of the swirl chamber and A_i is the inlet cross-sectional area. Taking $\gamma_{water} = 73\times10^{-3}$ kg/s^2 and $\rho_{water} = 1\times10^3$ kg/m^3, and again using the example from [2]: r_s = 5.6mm, A_i = 2.56mm^2, r_o = 1.0mm, r_{oac} = 0.79mm and Q = 4.1×10^{-5}m^3/s. This gives $\delta r = 1.1 \times 10^{-8}$ m. So the calculated air core, for this example, would be 1.1×10^{-8}m smaller, if surface tension is considered, i.e. of the order of 100,000th of r_{oac}. Therefore the overall effect of surface tension in diminishing the size of the air-core is minimal.

An alternative method of determining δr, is to consider the equivalent pressure, Δp_{st}, that would be required to oppose the surface tension force, for equilibrium. Consider the semi circle in Fig.(2), representing a half an air core. The surface tension force per unit length in the x-direction, into the paper, is given 2γ (kg/s^2). This must be balanced by the pressure force differential across the surface boundary and this is given by the pressure, Δp_{st}, acting upon the projected area of the semi-circle, $2r_{ac}$. Thus
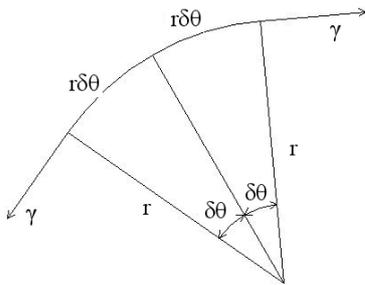
$$2\gamma = \Delta p_{st}\, 2r_{ac}. \qquad (6)$$

Figure 1
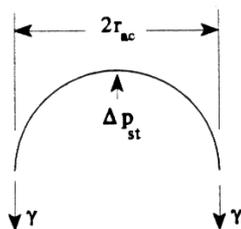

Figure 2

Rearranging Eqn.(6) and again using the example from [2] gives

$$\Delta p_{st} = \frac{\gamma}{r_{ac}} = \frac{73\times10^{-3}}{0.79\times10^{-3}} = 92 \text{ Pa}. \qquad (7)$$

The atmospheric pressure on the air core is $p_a \sim 0.1$ MPa, therefore Δp_{st} is indeed small in comparison. A formulation, based on the Bernoulli equation, of Abramovich [4] may be used to determine the equivalent increase in air core radius that this pressure, p_{st} would cause.

$$-p = \frac{\rho}{2}\left\{\frac{Q(r_s - r_i)}{A_i}\right\}^2 \left(\frac{1}{r^2} - \frac{1}{r_{oac}^2}\right), \quad \text{thus}$$

$$\tag{8}$$

$$-\Delta p_{st} = -\frac{\gamma}{r_{oac}} = \frac{\rho}{2}\left\{\frac{Q(r_s - r_i)}{A_i}\right\}^2 \left(\frac{1}{r^2} - \frac{1}{r_{oac}^2}\right).$$

Using the values of the from the example of [2] gives r = 0.000790008m therefore $\delta r = r - r_{oac} = 8 \times10^{-9}$m.
This is of a similar order to the value obtained for δr above. It may therefore be concluded that, to all intents and purposes, the surface tension affects on the surface of the air core of a swirl atomizer may safely be ignored.

Mach Number, M

The Mach number is the ratio of an existing velocity, of an object, a liquid or a gas, in a given gaseous environment to the speed of sound of the object, liquid or gas in the same gaseous environment. One normally associates the use of Mach number with nozzles in terms of converging-diverging nozzles using compressible gasses [5]. In terms of swirl atomizer liquid flows one conceivable supersonic application is the injection of liquid fuel in internal combustion (I.C.) engines. The time of the inlet stroke (t_{is}) on a four-cylinder, four-stroke I.C. engine, running at 4,000 R.P.M is

$$t_{is} = \frac{1}{2\times4000\times60} = 2.08 \text{ ms}. \qquad (9)$$

(there are two inlet strokes per revolution on a four-cylinder engine.) If this same engine is running at 60 M.P.H. and using 30 M.P.G. then volume of gasoline used (assuming an imperial gallon) per inlet stroke is

$$v_{is} = \frac{60\times4.54\times10^{-3}}{30\times60\times2\times4000} = 1.89\times10^{-8}\text{m}^3. \qquad (10)$$

If one assumes an orifice diameter of 0.9mm [6] then the minimum gasoline velocity is

$$u_{gasoline} = \frac{1.89\times10^{-8}}{2.08\times10^{-3}\times\left(0.45\times10^{-3}\right)^2\times\pi} = 14.28 \text{ m/s}. \qquad (11)$$

However, for a swirl atomizer injector, the gasoline flows only through a liquid anulus in the orifice. This annulus is reported [6] to be around a third of the orifice radius. Thus a truer axial gasoline velocity would be

$$u_{gasoline} = \frac{1.89 \times 10^{-8}}{2.08 \times 10^{-3} \times \left(0.45^2 - 0.3^2\right) \times 10^{-6} \times \pi} = 25.71 \text{ m/s.}$$

(12)

This order of magnitude agrees with the findings, from CFD modelling, of [6]. The injectors for the work of [6] were clearly designed with the above type of fuel usage figures from the outset. Incidentally, the similar sized small nozzles of [2] had this order of axial velocity.

The speed of sound in quiescent air at sea level is taken to be 340.29 m/s. However conditions within the cylinder of an I.C. engine are very different from this. The gaseous medium within the cylinder is, during the inlet process, a mixture of air and gasoline, not just air. This ratio of air to gasoline (approximately 11:1 for a warmed up engine) will likely remain fairly constant during the inlet process. This is because as the piston goes down and more air is drawn in, so to is more fuel injected. The gaseous mixture will therefore likely be approximated by an ideal gas. The walls of the cylinder will heat the incoming air and fuel, indeed the air will be pre-heated. The process will not therefore be adiabatic. The incomming air will not be compressed at this stage. The inlet process will therefore be isentropic.

To calculate the relative velocity of the injected liquid fuel with the air in the cylinder one needs to calculate the inlet speed of the air. Many contemporary four-stroke engines have four valves per cylinder and the average inlet air velocity will therefore concur with the speed of the piston. If one considers a piston of stroke 100mm at 4000 R.P.M. then its velocity will be

$$u_{piston} = \frac{0.1 \times 2 \times 4000}{60} = 13.\overline{3} \text{ m/s.}$$

(13)

Therefore, although high velocities are involved in the I.C. engine processes, relative velocities are low. Also vaporization of liquid fuel will take place quickly so that mixing occurs rather than relative motion. The mixing is dependent largely on the turbulent and heating effects more than the relative motion. Turbulence is due to rapid drawing in of air through the inlet ports. It is therefore unlikely that a meaningful mach number can be defined for the fuel injection process.

Swirl Number, S_0

One definition [7] of the swirl number is as the ratio of the axial flux of angular momentum to the product of the axial momentum flux and a characteristic radius, which may be taken as the outlet orifice radius r_o. The angular momentum flux is the axial mass flux (the volumetric flow rate, Q times the liquid density ρ, to give $Q\rho$) times the product of the angular (swirl or tangential) velocity, w and the radius, r. For low viscosity liquids the flow may be approximated by free-vortex flow [1] so that wr = constant. Thus

$$wr = w_i(r_s - r_i) = w_i R = c$$

(14)

where w_i is the tangential velocity at the inlets, r_s is the swirl chamber radius and r_i is the radius of the inlets. Therefore R = $r_s - r_i$ is the mean radius at which the flow enters the swirl chamber. The swirl number can be given by

$$S_0 = \frac{Q\rho wr}{Q\rho u_o r_o} = \frac{wr}{u_o r_o} = \frac{w_i R}{u_o r_o}.$$

(15)

The inlet tangential velocity can be approximated by dividing the volume flow, Q by the inlet cross-section, A_i and, in the absence of a knowledge of the air core diameter, the axial velocity at the outlet can be given by dividing the volume flow by the outlet cross-section area, πr_o^2

$$w_i = \frac{Q}{A_i} \quad \text{and} \quad u_o = \frac{Q}{\pi r_o^2}.$$

(16)

This allows Eqn.(15) to be written in the form given by [7]:

$$S_0 = \frac{\pi R r_o}{A_i}.$$

(17)

The swirl number is useful in determining the ratio of angular momentum to axial momentum. Note that S_0 in Eqn.(15) could have wr = c = $w_o r_o$ so that $S_0 = w_o/u_o$. However, as the swirl flow approximates a free vortex then simply picking a particular swirl velocity does not provide a useful measure and either Eqn.(17) and the right hand form of Eqn.(15) provides a useful measure for comparing the performance between different atomizers.

Reynolds Number, Re

The Reynolds number is usually defined as the ratio of inertial force to viscous force. (Re has also been defined as the ratio of dynamic pressure to shearing stress. The units of Re are the same for both) An inertial force, in terms of Newton's first law, is the force needed to alter the state of motion of an object or liquid, including if it is initially at rest. An inertial force can also be thought of as a rate of change of momentum:

$$\text{Inertial Force} = QV\rho = V^2 L^2 \rho.$$

(18)

The viscous drag force acting on an object or liquid is proportional to the viscosity, its velocity and a charecteristic length, L:

$$\text{Viscous Force} = \mu V L.$$

(19)

This characteristic length for pipe flow could be the diameter of the pipe; the smaller the diameter then the greater the proportion of the boundary layer and hence the greater the drag force. The characteristic length could also be the distance along a surface on which the liquid flows. Thus

$$Re = \frac{V^2 L^2 \rho}{\mu V L} = \frac{V L \rho}{\mu} = \frac{V L}{\nu}$$

(20)

where ν is the kinematic viscosity. The Reynolds number is essentially a measure of the two opposing forces; of the momentum acting to continue the flow against the retarding viscous forces. If the flow is able to acquire sufficient

velocity then it may become turbulent. The numerical value for the Reynolds number, for any particular type of flow, is then an indication of whether the flow is laminar or turbulent.

An estimate may be made as to whether the flow within a swirl atomizer is expected to be laminar or turbulent. In experiments on a particular atomizer by [8], using water, at an operating pressure of $\Delta p = 10.364$ MPa gauge, the volumetric flow rate was $Q = 5.683 \times 10^{-5}$ m³/s. The mean inlet velocity w_i may be calculated by dividing the volumetric flow rate by the cross-sectional area of the two $r_i = 0.6$ m.m. inlets to give $w_i = 25.125$ m/s. The Reynolds number for the 'pipe flow' through the tubular inlets is

$$Re = \frac{w_i 2r_i}{\nu} = \frac{25.125 \times 1.2 \times 10^{-3}}{1 \times 10^{-6}} = 30,150. \qquad (21)$$

As described by [9], for example, pipe flow should be turbulent for $Re > 2,300$. However:

(1) The tubular inlets are short, $l_i = 2.4$ mm, so that fully developed turbulence may not be likely to occur before entering the swirl chamber.

(2) Whether or not turbulence occurs within the inlets there is no guarantee of turbulence occurring, or being maintained, within the body of the atomizer. This is because the radial forces in the swirling liquid tend to laminarise the flow.

It is informative to consider the state of this swirling flow. The average axial velocity across the swirl chamber cross-section is equal to the volumetric flow rate Q divided by the swirl chamber cross-sectional area $\pi \times \left(4.25 \times 10^{-3}\right)^2$ giving $u = 1$ m/s (ignoring the presence of the air-core). Thus the ratio of swirl to axial velocity, at the inlet level is approximately 25:1 and therefore the swirl velocity very much dominates the flow. Although some work has been carried out on the modelling of turbulent swirling confined flows, e.g. [10], as far as is known an investigation has not been carried out to determine the criteria for the transition to turbulence for these flows and indeed such criteria may differ between flow types. For instance the criteria for the onset of turbulence in the furnace and jet-engine combustors considered by [10] may differ somewhat from those for swirl atomizers. However, some quantitative appreciation of the Reynolds number for the onset of turbulence may be gained with reference to the early work of [11, 12] who carried out investigations into the transition to turbulence on concave walls and suggested that transition to turbulence occurs within the boundary layer for

$$\frac{V_\infty \delta_2}{\nu} \sqrt{\frac{\delta_2}{r}} > 7 \qquad (22)$$

where V_∞ is the free stream velocity, r is the radius of curvature of the wall and δ_2 is the momentum thickness within the near wall boundary layer. According to [13] the momentum thickness for flows along a concave wall is given by

$$\delta_2 = 0.047\delta \quad \text{where} \quad \delta = \left(\frac{2\nu\xi}{V_\infty}\right)^{1/2} \qquad (23)$$

is the boundary layer thickness and ξ is the distance along the wall at which the transition to turbulence takes place. Eqn.(22) is akin to the transition number for Taylor-Couette flow, between two cylinders with the inner one rotating,

$$T_a = \frac{w_{in}d}{\nu} \sqrt{\frac{d}{R_{in}}} > 41.3 \qquad (24)$$

for turbulent flow, where w_{in} is the tangential velocity of the inner cylinder, R_{in} is its radius and d is the radial separation between the inner and outer cylinders. A combination of eqns.(22) and (23) gives, for transition to occur,

$$Re = \frac{V_\infty \xi^3}{\nu r^2} > 2.8 \times 10^{10}. \qquad (25)$$

Eqn.(25) may be used to estimate whether or not the flow within the atomizer of [8] is turbulent. If one equates $w_i = 25$ m/s with V_∞ and equates $r_s = 4.25$ mm with r, and with $\nu = 1 \times 10^{-6}$ m²/s for water, then it will be found that $\xi > 0.27$ m for turbulence to occur. This is equivalent to a fluid particle, that enters at the inlet, making some ten revolutions of the 8.5 mm diameter swirl chamber. With the mean tangential and axial velocities of 25 m/s and 1 m/s, respectively, then the particle, near the wall, will actually undergo some twenty revolutions during its traverse through the $l_s = 21.25$ mm swirl chamber. This suggests that turbulence may occur, at least within the swirl chamber boundary layer and at the very high liquid pressure used by [8].

A more fundamental analysis lends weight to the above. According to [14] the influence of wall curvature on transition is small if $\delta/|r| \ll 1$. In other words, the wall curvature is such that the flow approximates that of a flat plate. For flow along a plane wall the boundary layer thickness may be approximated [15] by

$$\delta = 5\sqrt{\frac{\nu \xi}{V_\infty}}. \qquad (26)$$

With $w_i = 25$ m/s equated with V_∞ and $r_s = 4.25$ mm equated with r, then eqn.(26), gives

$$\frac{\delta}{|r|} = \frac{5}{4.25 \times 10^{-3}} \sqrt{\frac{1 \times 10^{-6} \xi}{25}} = 0.235\sqrt{\xi}. \qquad (27)$$

This is much less than unity for small ξ. Thus the swirl flow along the wall of the swirl chamber may be approximated by that along a plane wall. The critical Reynolds number for flow along a plane wall is

$$Re = \frac{V_\infty \xi}{\nu} = 3.2 \times 10^5. \qquad (28)$$

If one again equates $w_i = 25$ m/s with V_∞ then it will be seen that ξ only need be greater than about 13 mm for turbulence to occur. Comparison of eqns.(23) and (26) for δ show that the laminar boundary layer for flow along plane walls is approximately 3.5 times that for concave walls. This was explained by [10], who refer to a 'body-force' effect due to the centrifugal acceleration which effectively forces the

liquid towards the wall and hence reduces the boundary layer thickness. This same effect also has a tendency to restrict the diffusion of turbulent fluctuations in the radial direction. The swirl atomizer considered here used an unusually high liquid velocity and also had an unusually long swirl chamber. These two conditions combined to give a likelihood of turbulent flow, at least at the end of the swirl chamber. Swirl atomizers operating at lower inlet Reynolds numbers, and particularly with shorter chambers, are unlikely to develop turbulent flow. For example swirl atomizers employed in aerosols are approximately of a quarter of the size of the one considered and will have volumetric flow rates of perhaps 1×10^{-6} m^3/s. If these are operated with liquids of the constituency of water, $v_L = 1 \times 10^{-6}$ m^2/s, then on consideration of their operational Reynolds number from eqn.(25), the distance, ξ, that a fluid particle would be required to traverse within the flow domain, in order for turbulence to occur, would be approximately 0.2m and this is clearly much larger than the order of scale of such nozzles.

Weber Number, We, on a Droplet

One may use the notion of a Weber number, eqn.(4), in order to aid in understanding the, inertial, aerodynamic force acting on a droplet in comparison to the surface tension force. The integrity of the drop is maintained by the surface tension force acting against the internal pressure of the droplet. The difference in the internal pressure and the ambient, atmospheric pressure is $p_i - p_a$. The inertial aerodynamic force further reduces the pressure acting on the outside surface of the droplet to p_s, where $p_s < p_a$. The inertial force acting on the droplet is then akin to aerodynamic lift, fig 3.
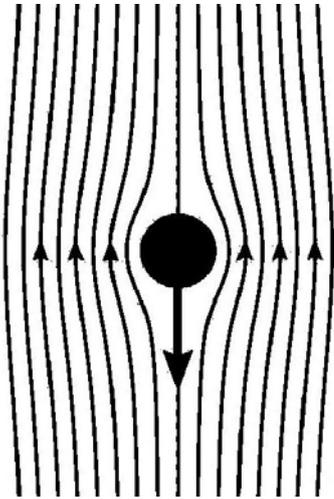

Figure 3

The pressure difference between the ambient pressure, p_a and the pressure on the surface of the droplet, p_s can be computed using the Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} = \text{Constant}. \tag{29}$$

If it takes the drop some arbitrary time t to traverse its diameter D then its velocity is V = D/t. At the same time the air flow around the drop has to travel a distance $\pi D/2$, so that

the velocity of the air flowing around the droplet is $\pi D/2t$. Bernoulli then gives

$$\frac{p_a}{\rho} + \frac{\left(\frac{D}{t}\right)^2}{2} = \frac{p_s}{\rho} + \frac{\left(\frac{\pi D}{2t}\right)^2}{2} \tag{30}$$

Therefore

$$p_a - p_s = \frac{\rho}{2}\left(\frac{D}{t}\right)^2 \left(\frac{\pi^2}{2^2} - 1\right) \tag{31}$$

or

$$p_a - p_s = \frac{\rho}{8} V^2 \left(\pi^2 - 4\right). \tag{32}$$

The inertial force acting to pull on the side of the droplet is this pressure differential applied onto the projected area of the drop

$$\text{Inertial Force} = (p_a - p_s)\pi\left(\frac{D}{2}\right)^2 = \frac{\rho}{8}V^2\left(\pi^2 - 4\right)\pi\left(\frac{D}{2}\right)^2. \tag{33}$$

Opposing this, acting to retain the integrity of the drop, is the surface tension force, This is simply the surface tension force per unit length, γ times the perimeter, fig. 4,

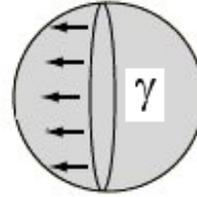$$\text{Suface Tension Force} = \gamma\pi D. \tag{34}$$

Figure 4

Therefore a Weber number providing a ratio of these two forces is

$$\text{We} = \frac{\frac{\rho}{8}V^2\left(\pi^2 - 4\right)\pi\left(\frac{D}{2}\right)^2}{\pi D\gamma} = \frac{\rho V^2\left(\pi^2 - 4\right)D}{32\gamma}. \tag{35}$$

As indicated above, even in the quiescent condition, the surface tension is maintaining the integrity of the droplet against the internal pressure, p_i. The force acting on a projected area of the quiescent droplet, due to this internal pressure, is the difference between internal pressure and ambient pressure times the projected area

$$F_{\text{internal}} = (p_i - p_a)\pi\left(\frac{D}{2}\right)^2. \tag{36}$$

Balancing this force is the force due to the surface tension, given in eqn.(34), as before. Therefore, for equilibrium, in the quiescent state

$$\left(p_i - p_a\right)\pi\left(\frac{D}{2}\right)^2 = \gamma\pi D \quad \text{or} \quad p_i - p_a = \frac{4\gamma}{D}. \qquad (38)$$

This result can also be found by considering the radius of curvature [16]. Thus the entire pressure differential acting, across the droplet, from the intrinsic internal pressure to the surface pressure, reduced by aerodynamic effects is, from eqns.(33) and (38),

$$p_i - p_s = \left(p_i - p_a\right) + \left(p_a - p_s\right) = \frac{4\gamma}{D} + \frac{\rho}{8}V^2\left(\pi^2 - 4\right). \qquad (39)$$

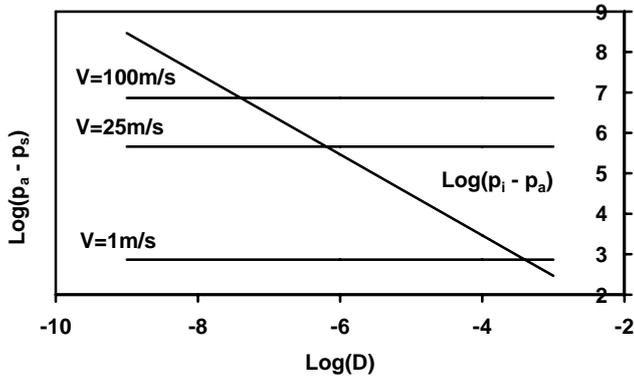| Dia	$p_i - p_a$	$p_a - p_s$	$p_a - p_s$	$p_a - p_s$
		V=1m/2	V=25m/s	V=100m/s
1mm	2.9E+02	7.3E+02	4.6E+05	7.3E+06
0.1mm	2.9E+03	7.3E+02	4.6E+05	7.3E+06
1μm	2.9E+05	7.3E+02	4.6E+05	7.3E+06
10nm	2.9E+08	7.3E+02	4.6E+05	7.3E+06

Table 1

Figure 5

The two different pressure differentials are tabulated in Table 1 and plotted in fig 5 for water droplets at STP, for three different velocities. In this simple treatment the aerodynamic pressure difference is only dependent on the velocity. The surface tension pressure increases exponentially as D decreases. Thus the break up of droplets requires an exponential increase in kinetic energy in relation to drop diameter. It has recently been discovered, [17], perhaps not surprisingly, that drops of water fail to 'splash' when in a vacuum. Tabe 1 indicates that with very small droplets the surface tension force may still be dominant within a vacuum, and aerodynamic break up effects would be negligible or non existent.

Summary

There is probably no definitive list of numeric's, or dimensionless quantities, for swirl atomizers. Certainly [18] covers many. The purpose of the present article has been to provide a novel view of several numeric's and to use them as a means to discuss several aspects of swirl atomizer flow behaviour.

Nomenclature

A_i Cross-sectional area of the Inlets ($A_i = i\pi r_i^2$ for circular inlets)
c   Free Vortex constant (wr = c)
Fr   Froude Number
g   Gravity
h   Head or Height
L   Characteristic length
l_i Length of inlets
l_s Length of swirl chamber
P   Pressure
P_a Ambient or Atmospheric Pressure
P_i Internal Pressure of a Drop
P_s Surface Pressure on a moving drop
P_{st} Static Pressure
ΔP Operating pressure, across atomizer
Q   Volumetric flow rate
R   Distance from axis of atomizer to centre of inlets ($R = r_s - r_i$)
Re   Reynolds number
r   Radius or radial direction
r_i Radius of round inlets
r_o Radius of the outlet (discharge orifice)
r_{ac} Radius of the air-core
r_{oac} Radius of the air-core in the outlet
r_s Radius of the swirl chamber
S_0 Swirl Number
t_{is} Time for inlet stroke
u   Axial Velocity
$u_{gasoline}$ Velocity of injected fuel
u_{piston} Velocity of piston
v_{is} Velocity of inlet stroke
V   General velocity
V_∞ Free stream velocity
w   Swirl Velocity
We   Weber Number
x   Axial Direction

Greek
δ Boundary Layer thickness, or small change
γ Surface Tension
μ Liquid Viscosity
ν Kinematic liquid viscosity ($\nu = \mu/\rho$)
ξ Distance along wall used in boundary layer and turbulence calculations
ρ Liquid Density
θ Angular direction

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